Fractals

Outcomes

Reusing this material

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The Mandelbrot Set

- The Mandelbrot Set is the set of numbers resulting from repeated iterations of the complex function:

$Z_n = Z_{n-1}^2 + C$ with the initial condition $Z_0 = 0$

- $C = x_0 + iy_0$ belongs to the Mandelbrot set if $|Z|$ converges.

$Z = x + iy \rightarrow Z^2 = x^2 + i2xy - y^2$

The Mandelbrot Set cont.

- Separating out the real and imaginary parts gives:

$$Z^r = x^2 - y^2 + x_0$$

$$Z^i = 2xy + y_0$$

- Take the threshold value as:

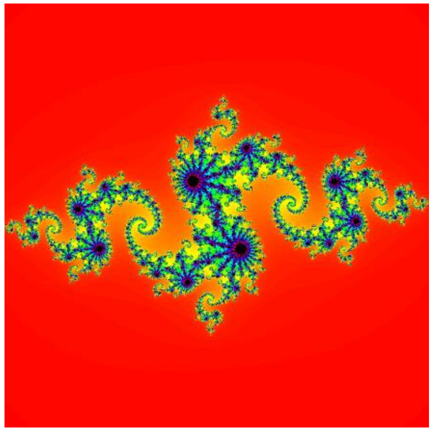
$$|Z|^2 \geq 4.0$$

- Set the maximum number of iterations N
  - Assume that z does not diverge at higher values of N

The Julia Set

- Similar algorithm to Mandelbrot Set

- Starting coordinates x_0 and y_0 represent fixed point from inside the Mandelbrot set



Visualisation

To visualise a Mandelbrot/Julia set:

- Represent the complex plane as a 2D grid.
- Calculate number of iterations N for complex numbers C corresponding to points on the grid.
- Convert the value of N to a colour and plot this on the grid.

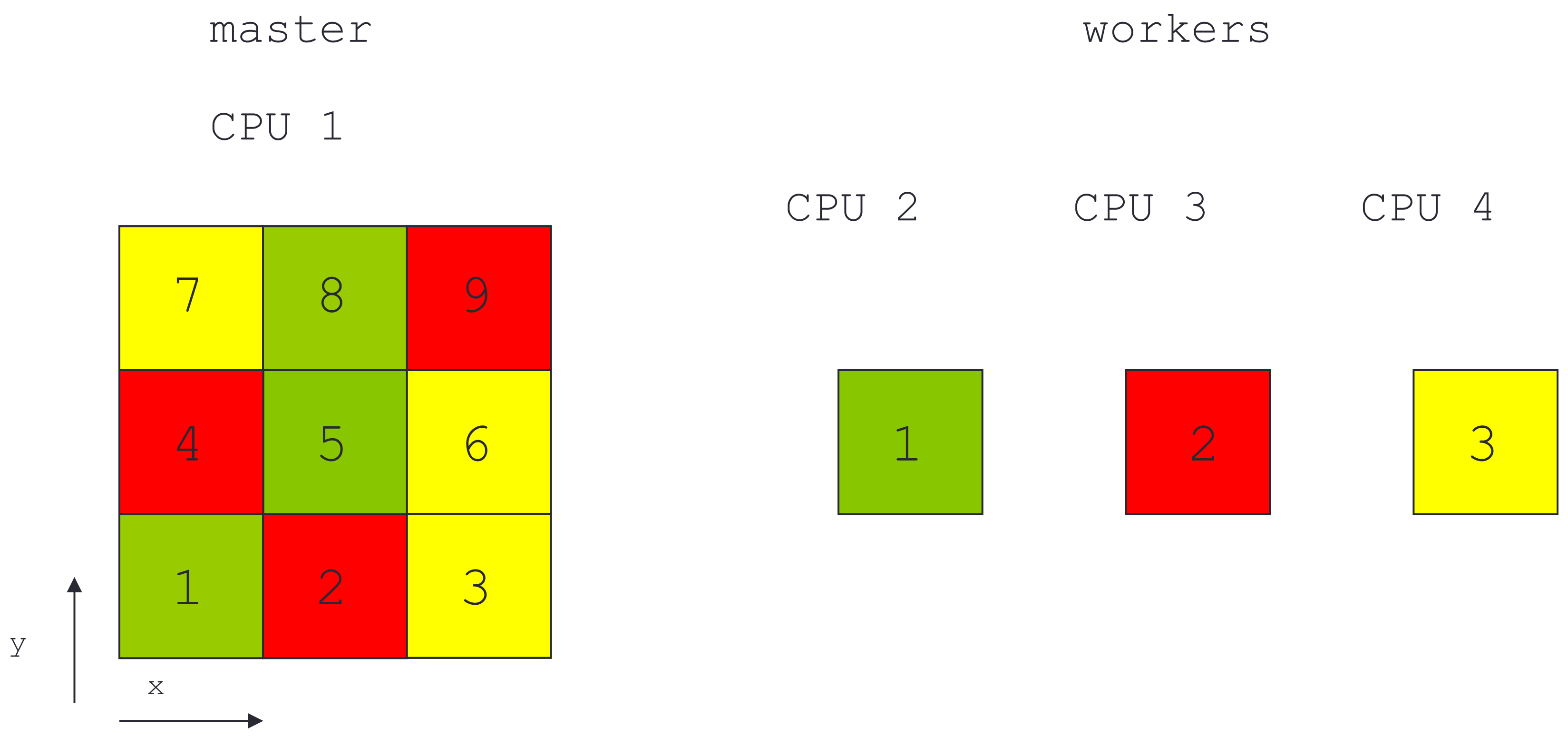
Parallelisation

- Values for each coordinate depend only on the previous values at that coordinate.
  - decompose 2D grid into equally sized blocks
  - no communications between blocks needed.
- Don't know in advance how much work is needed.
  - number of iterations across the blocks varies.
  - work dynamically assigned to workers as they become available.

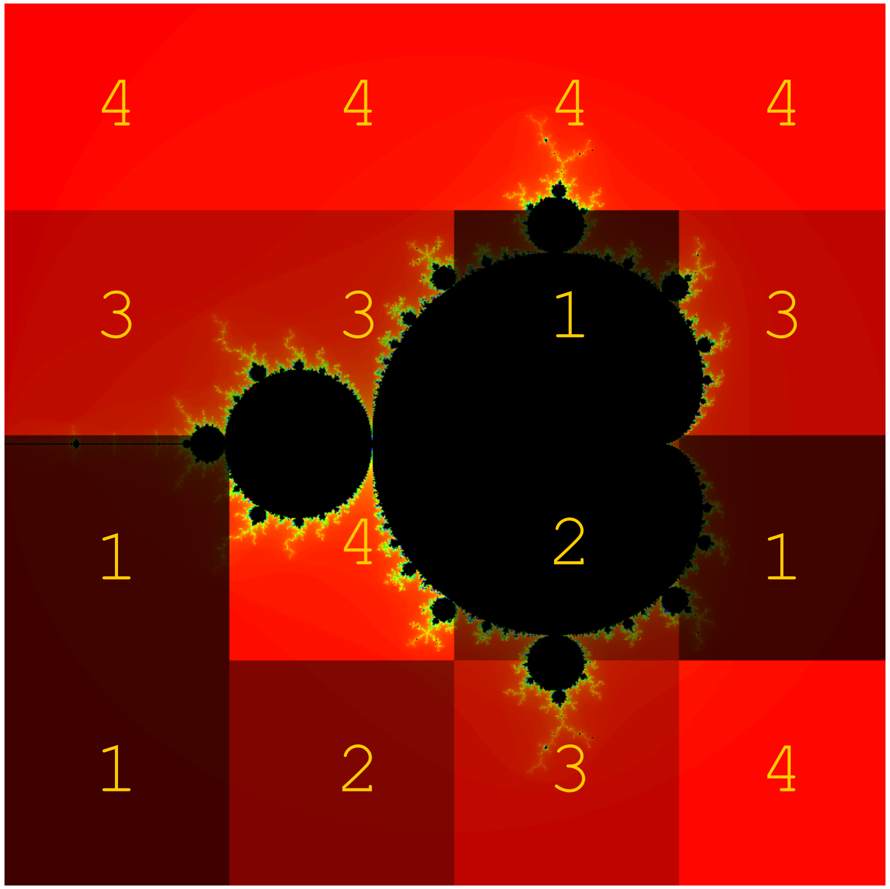
Implementation

- Split the grid into blocks:
  - each block corresponds to a task.
  - **master** process hands out tasks to **worker** processes.
  - workers return completed task to master.

Example: Parallelisation on 4 CPUs

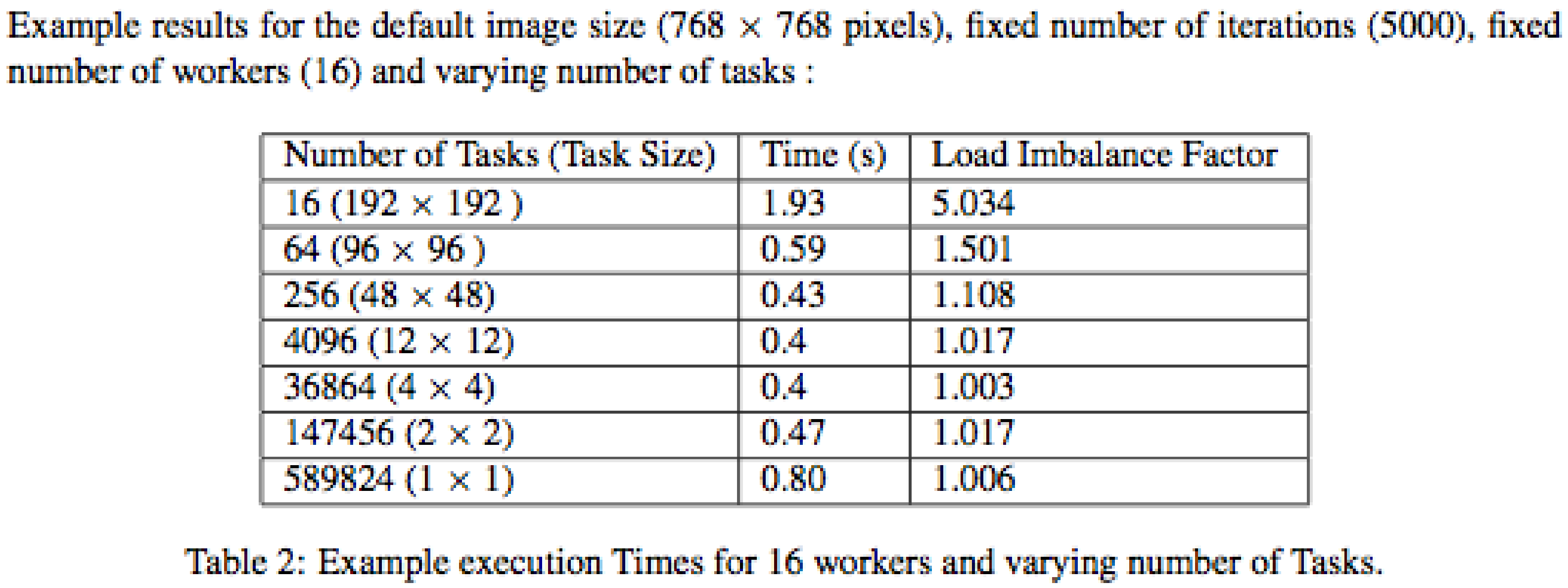


Parallelisation cont.



- taskfarm run on 5 CPUs
  - 1 master
  - 4 workers
- total number of tasks = 16

Example results – fixed number of workers

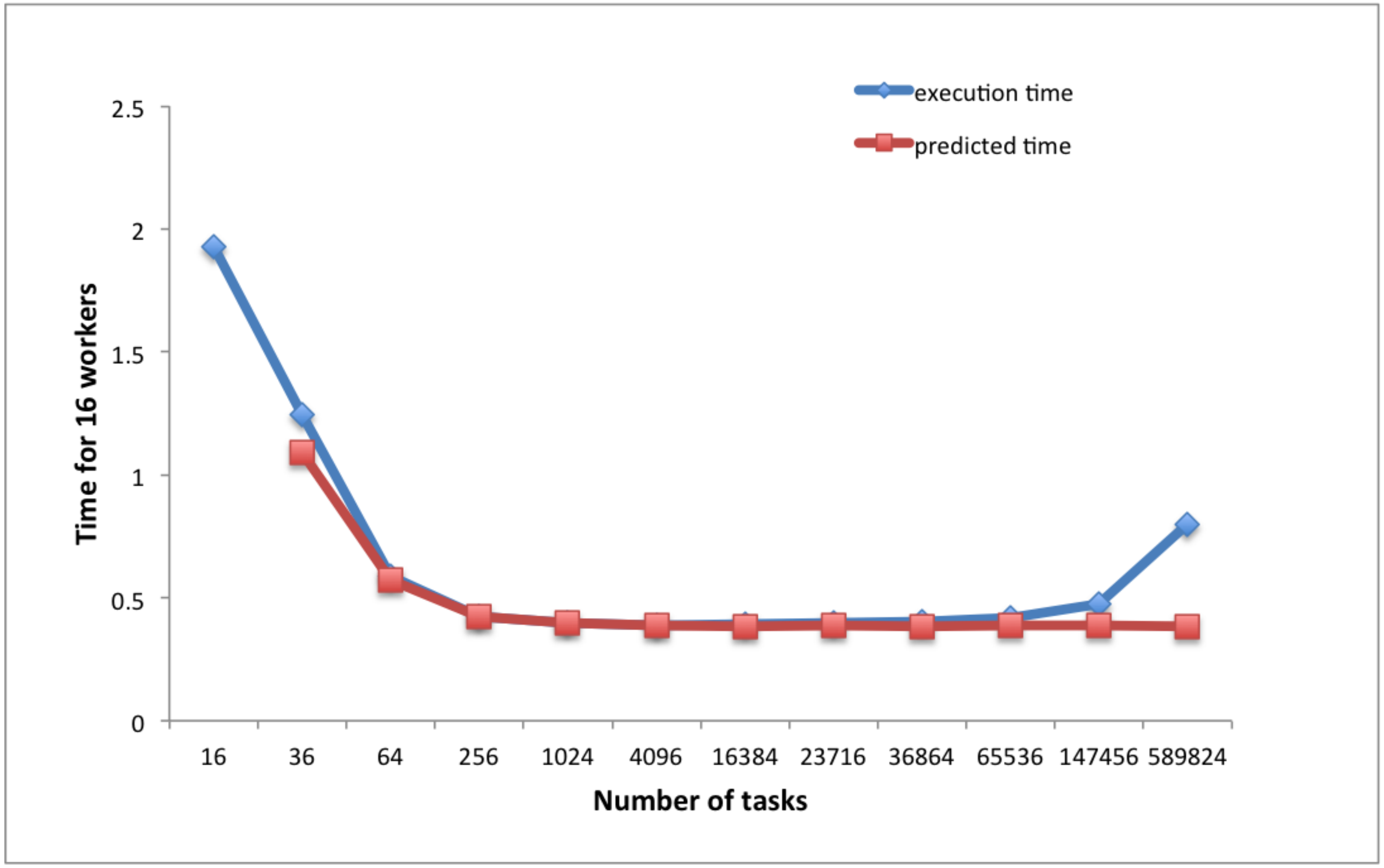


Example results for the default image size (768 × 768 pixels), fixed number of iterations (5000), fixed number of workers (16) and varying number of tasks :

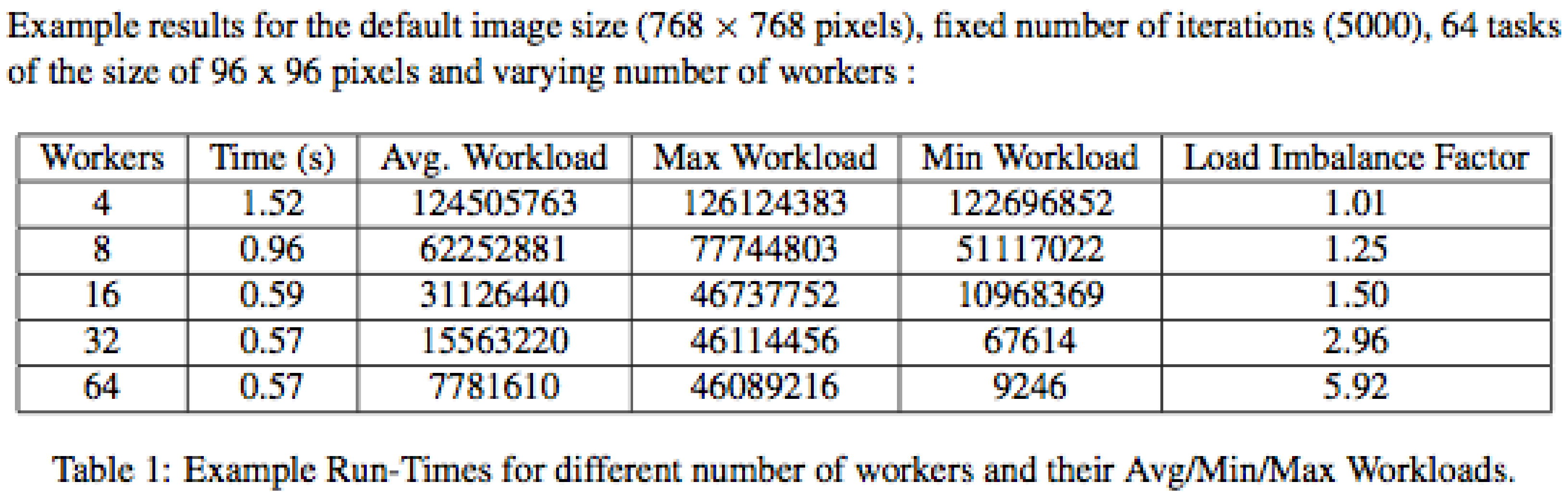
Number of Tasks (Task Size)	Time (s)	Load Imbalance Factor
16 (192 × 192)	1.93	5.034
64 (96 × 96)	0.59	1.501
256 (48 × 48)	0.43	1.108
4096 (12 × 12)	0.4	1.017
36864 (4 × 4)	0.4	1.003
147456 (2 × 2)	0.47	1.017
589824 (1 × 1)	0.80	1.006

Table 2: Example execution Times for 16 workers and varying number of Tasks.

Results cont.



Example results – fixed number of tasks

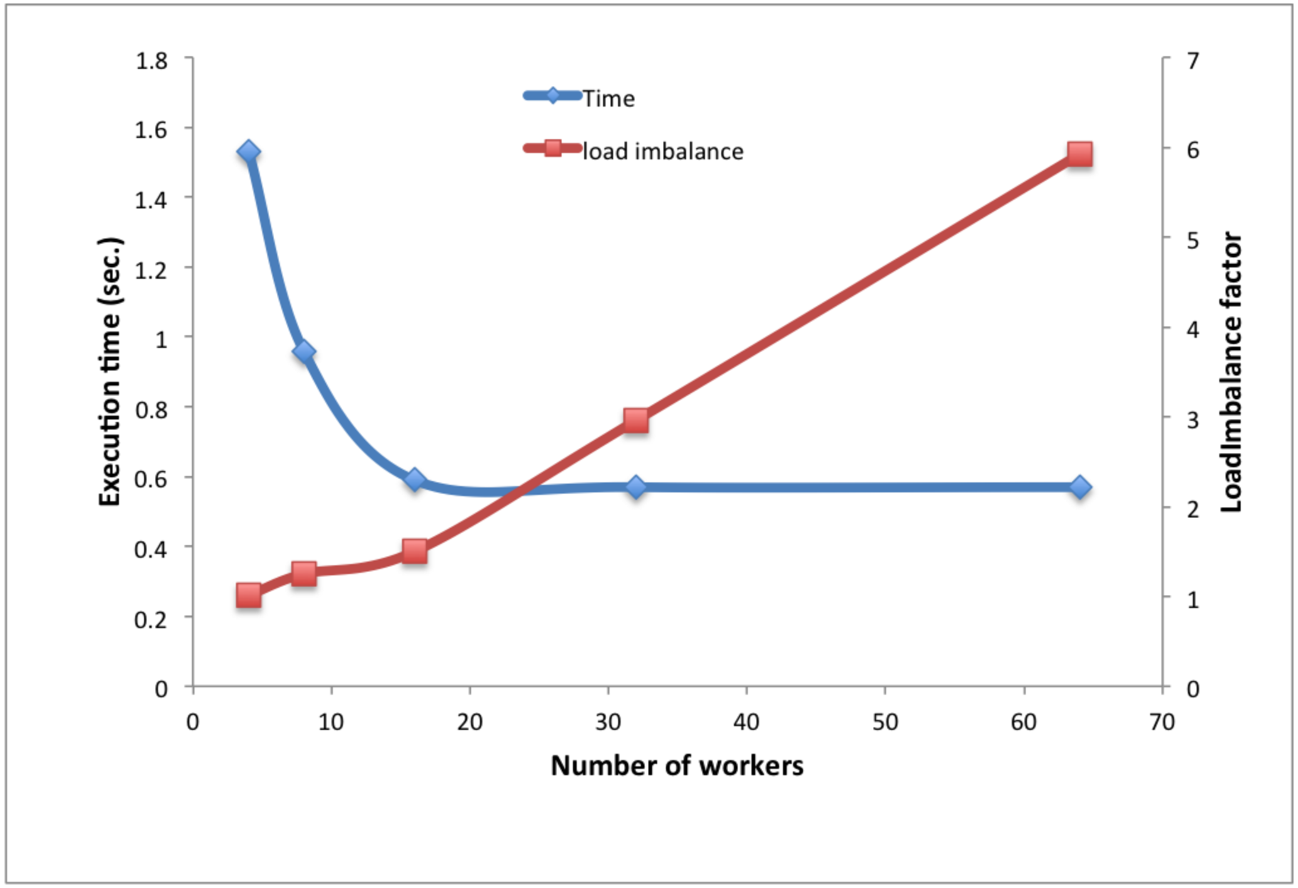


Example results for the default image size (768 × 768 pixels), fixed number of iterations (5000), 64 tasks of the size of 96 x 96 pixels and varying number of workers :

Workers	Time (s)	Avg. Workload	Max Workload	Min Workload	Load Imbalance Factor
4	1.52	124505763	126124383	122696852	1.01
8	0.96	62252881	77744803	51117022	1.25
16	0.59	31126440	46737752	10968369	1.50
32	0.57	15563220	46114456	67614	2.96
64	0.57	7781610	46089216	9246	5.92

Table 1: Example Run-Times for different number of workers and their Avg/Min/Max Workloads.

Results cont.



Key points to take away

TASK FARMS

- Also known as the master/worker pattern
- Allows a master process to distribute work to a set of workers processes.
- Can be used for other types of tasks but it complicates the situation and other patterns may be more suitable for implementing.
- Master process is responsible for creating, distributing and gathering the individual jobs.

Key points to take away

TASKS

- Units of work
- Vary in size, do not have to be of consistent execution time. If execution times are known it can help with load balancing.

QUEUES

- Master generates a pool of tasks and puts them in a queue
- Workers assigned task from queue when idle

Key points to take away

LOAD BALANCING

- How a system determines how work or tasks are distributed across workers (processes or threads)
- Successful load balancing avoids idle processes and overloading single cores
- Poor load balancing leads to under-utilised cores, reducing performance.

Key points to take away

COST

- Increasingly important
- Finite budgets require optimal use of resources requested.
- Load balancing is just one method of ensuring optimal usage and avoiding wasting resources.
- More power and resources do not necessarily mean improved performance.
- Always ask – is it necessary to run this on 4000 cores or could it be run on 2000 more efficiently?